Structural Analysis

Influence Line

Influence Line

It is the diagram showing the variation in the shear, moment, reaction, stress in a truss member or any other direct function due to a unit load moving across the structure. An influence line is constructed by plotting direct under the point, where the unit load is placed. An ordinate the height of which represents to some scale the value of the particular function being studied when the load is in that position.

For determinate structures influence lines are always straight lines.

Influence Line

- ***First***>>> Decide how the stress is to be calculated.
- ***Second***>> Move the unit load along the span and observe the effect on the stress in question.
- ***Third***>>> Calculate the controlling ordinates on the influence line and complete the diagram.

Influence Line of Beam

$$\sum M_B = 0; \left(clockwise, +ve\right)$$

$$R_A \times L - 1\left(L - x\right) = 0$$

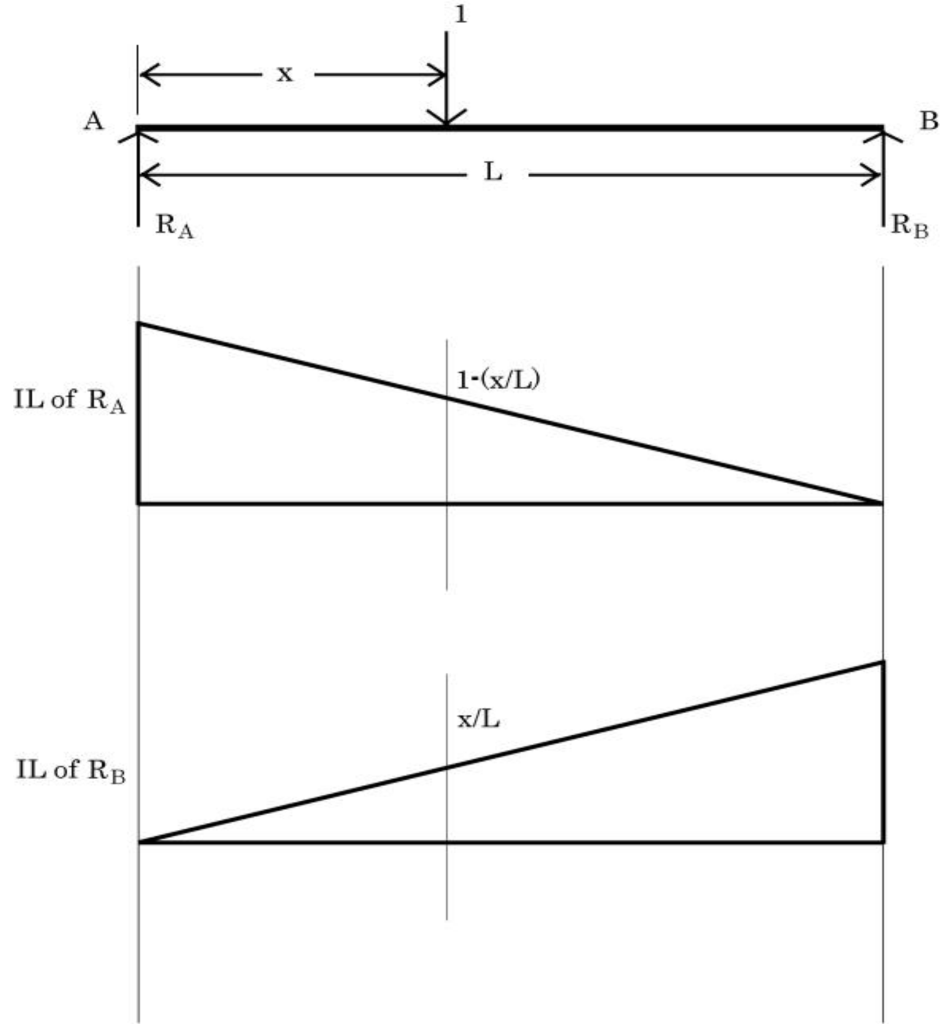
$$\therefore R_A = \frac{L - x}{L} = 1 - \frac{x}{L}$$

$$\sum V = 0; \left(upward, +ve\right)$$

$$R_A + R_B - 1 = 0$$

$$\Rightarrow 1 - \frac{x}{L} + R_B - 1 = 0$$

$$\therefore R_B = \frac{x}{L}$$

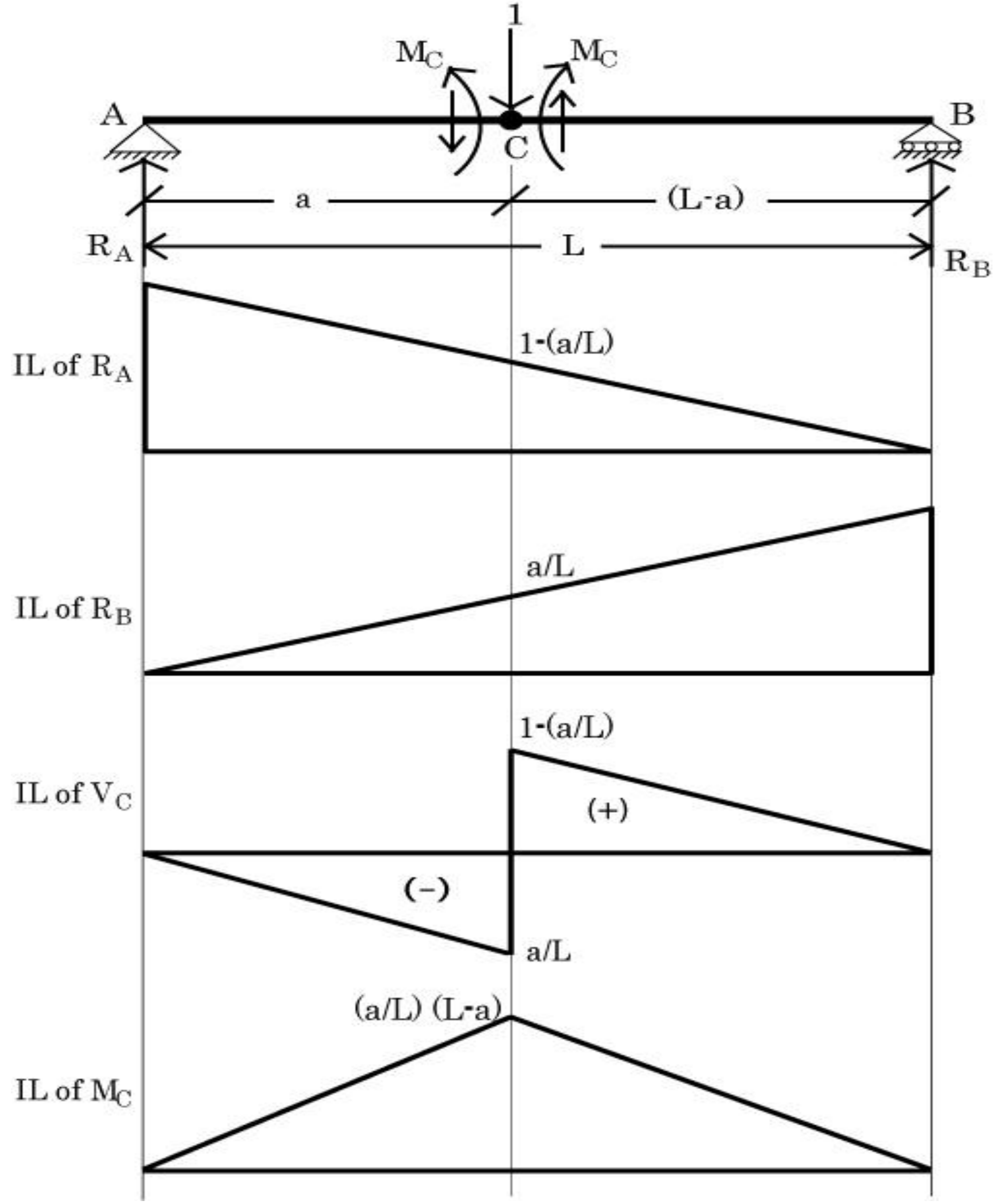


$$\sum M_A = 0; (clockwise, +ve)$$

$$1 \times a - R_B \times L = 0$$

$$\therefore R_B = \frac{a}{L}$$

$$and, R_A = 1 - \frac{a}{L}$$



IL of V_C:

When load at left side of point C;

$$\sum V = 0; (upward, +ve)$$

$$V_C + R_B = 0$$

$$\therefore V_C = -R_B$$

When load at right side of point C;

$$\sum V = 0; (upward, +ve)$$

$$-V_C + R_A = 0$$

$$\therefore V_C = R_A$$

IL of M_C:

When load at left side of point C;

$$\sum M_C = 0; (clockwise, +ve)$$

$$M_C - R_B(L-a) = 0$$

$$\Rightarrow M_C = R_B(L-a)$$

$$\therefore M_C = \frac{a}{L}(L-a)$$

When load at right side of point C;

$$\sum M_C = 0; (clockwise, +ve)$$

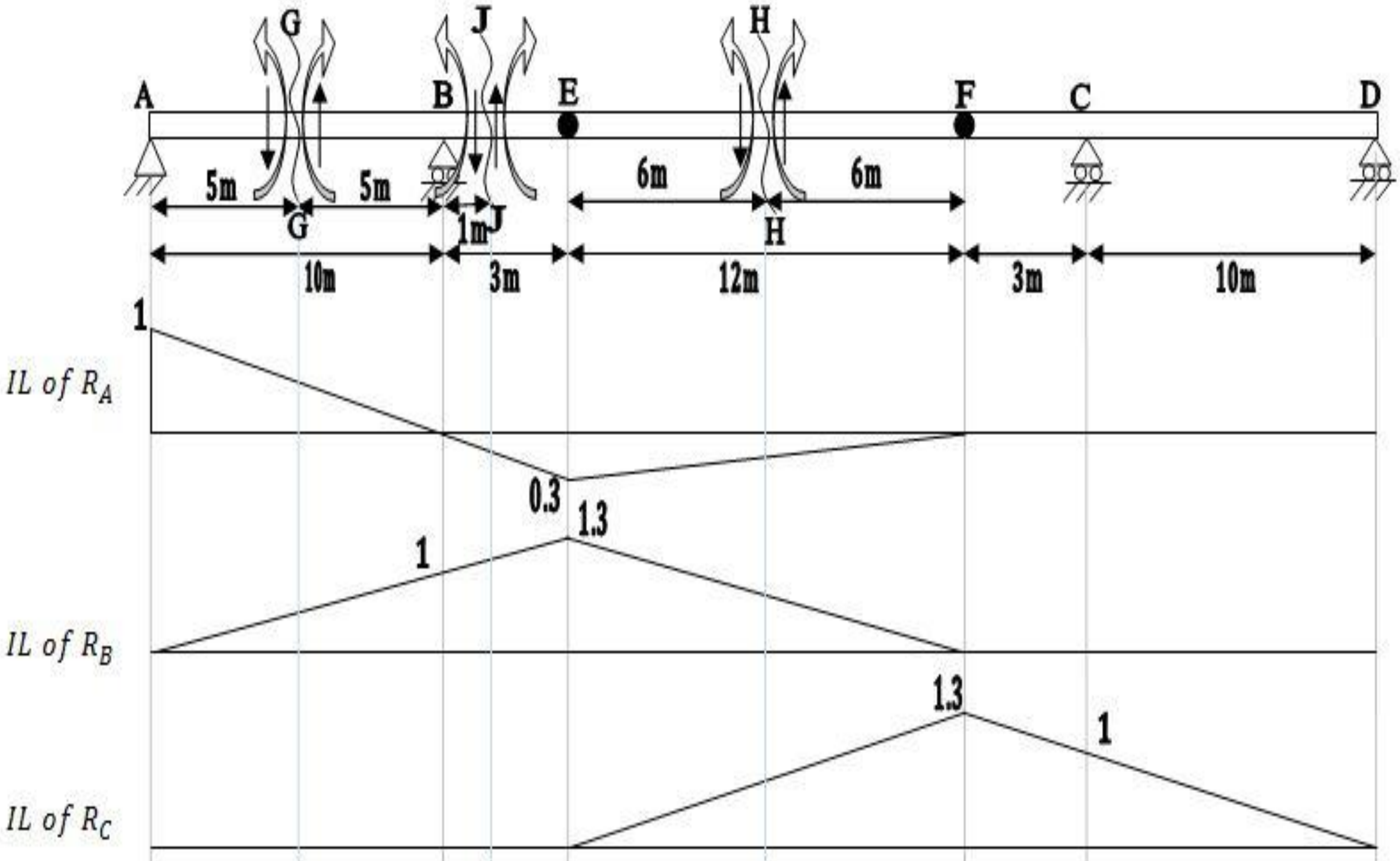
$$-M_C + R_A \times a = 0$$

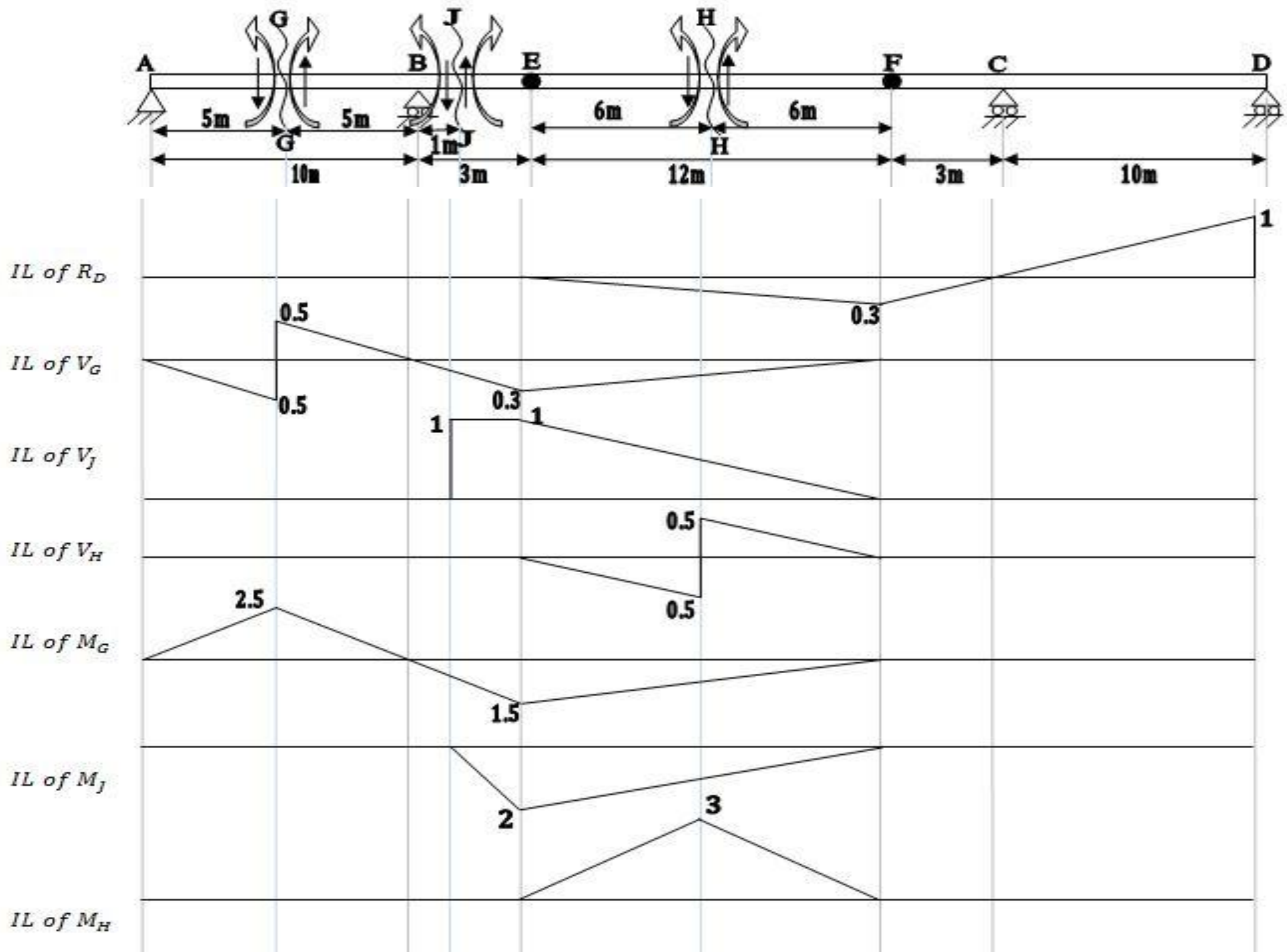
$$\Rightarrow M_C = aR_B$$

$$\therefore M_C = a\left(1 - \frac{a}{L}\right) = \frac{a}{L}(L-a)$$

Muller-Breslau Principle :

The ordinates of an influence line for any force are proportional to the deflected shape of the structure produce by removing the capacity of the structure to carry the force and then introducing into the modified structure. A displacement the corresponds to the restrained removed.


G
J
H
A
B
E
F
C
D
5m
5m
6m
6m
G
1m
J
H
10m
3m
12m
3m
10m
1
IL of R_A
0.3
1.3
1
IL of R_B
1.3
1
IL of R_C

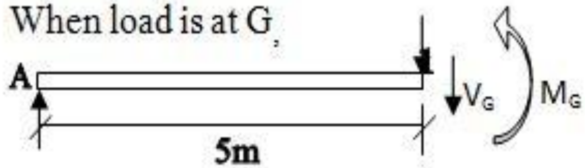


A
G
B
J
E
H
F
C
D
5m
5m
1m
6m
6m
10m
3m
12m
3m
10m
IL of R_D
1
0.3
IL of V_G
0.5
0.5
0.3
IL of V_J
1
1
IL of V_H
0.5
0.5
IL of M_G
2.5
1.5
IL of M_J
2
3
IL of M_H

Calculation

IL of M_G

When load is at G,



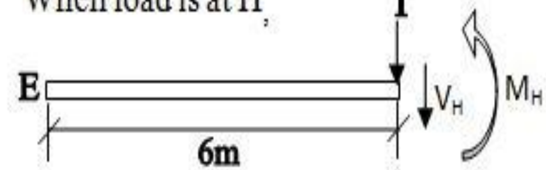
$\therefore \sum M_G = 0$ (+)

$0.5*5 - M_G = 0$

$\therefore M_G = 2.5$ KN-m

IL of M_H

When load is at H,



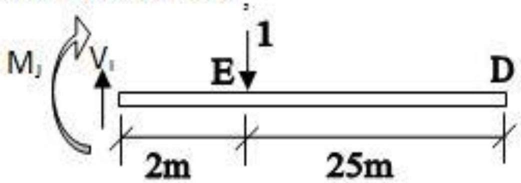
$\therefore \sum M_H = 0$ (+)

$0.5*6 - M_H = 0$

$\therefore M_H = 3$ KN-m

IL of M_J

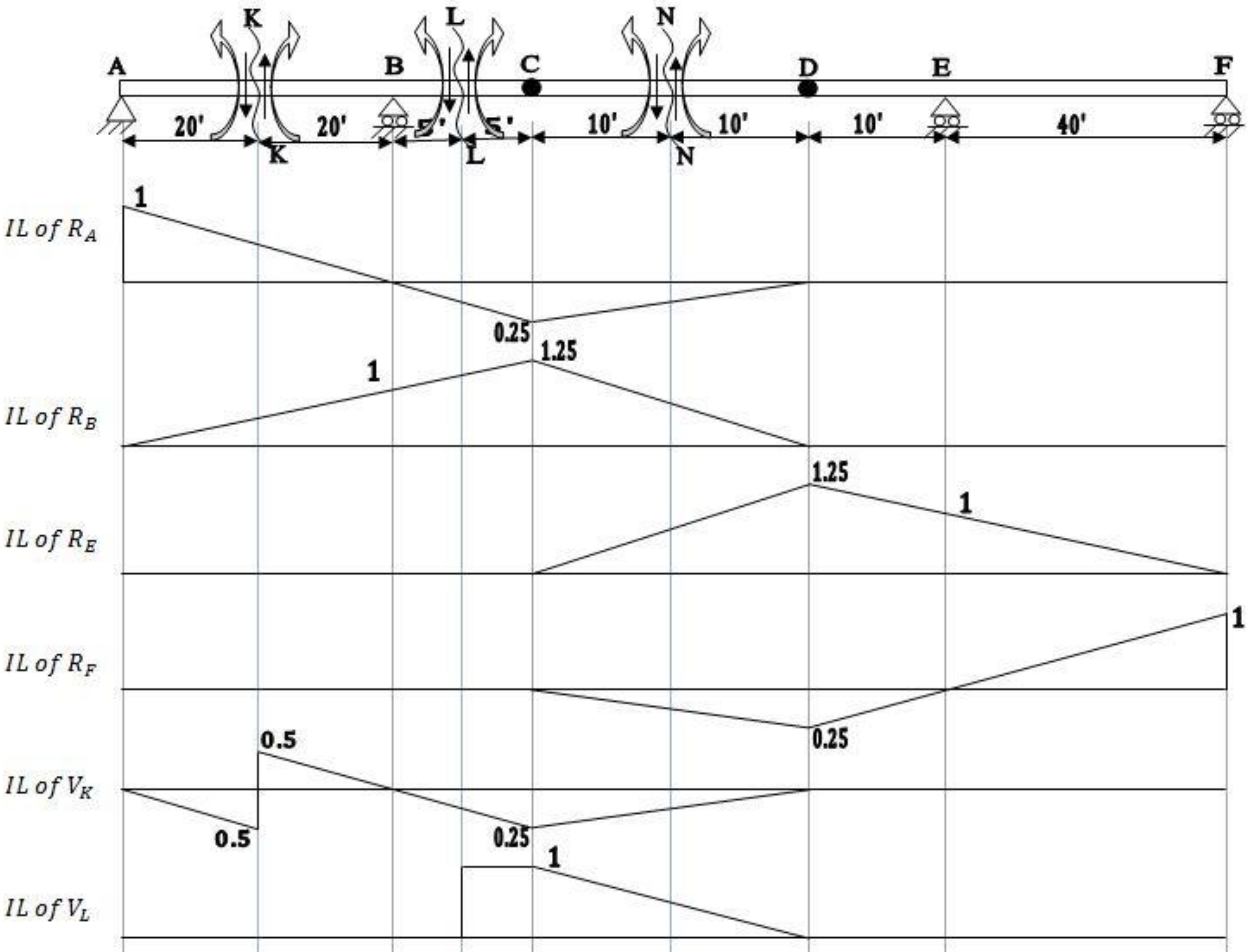
When load is at E,

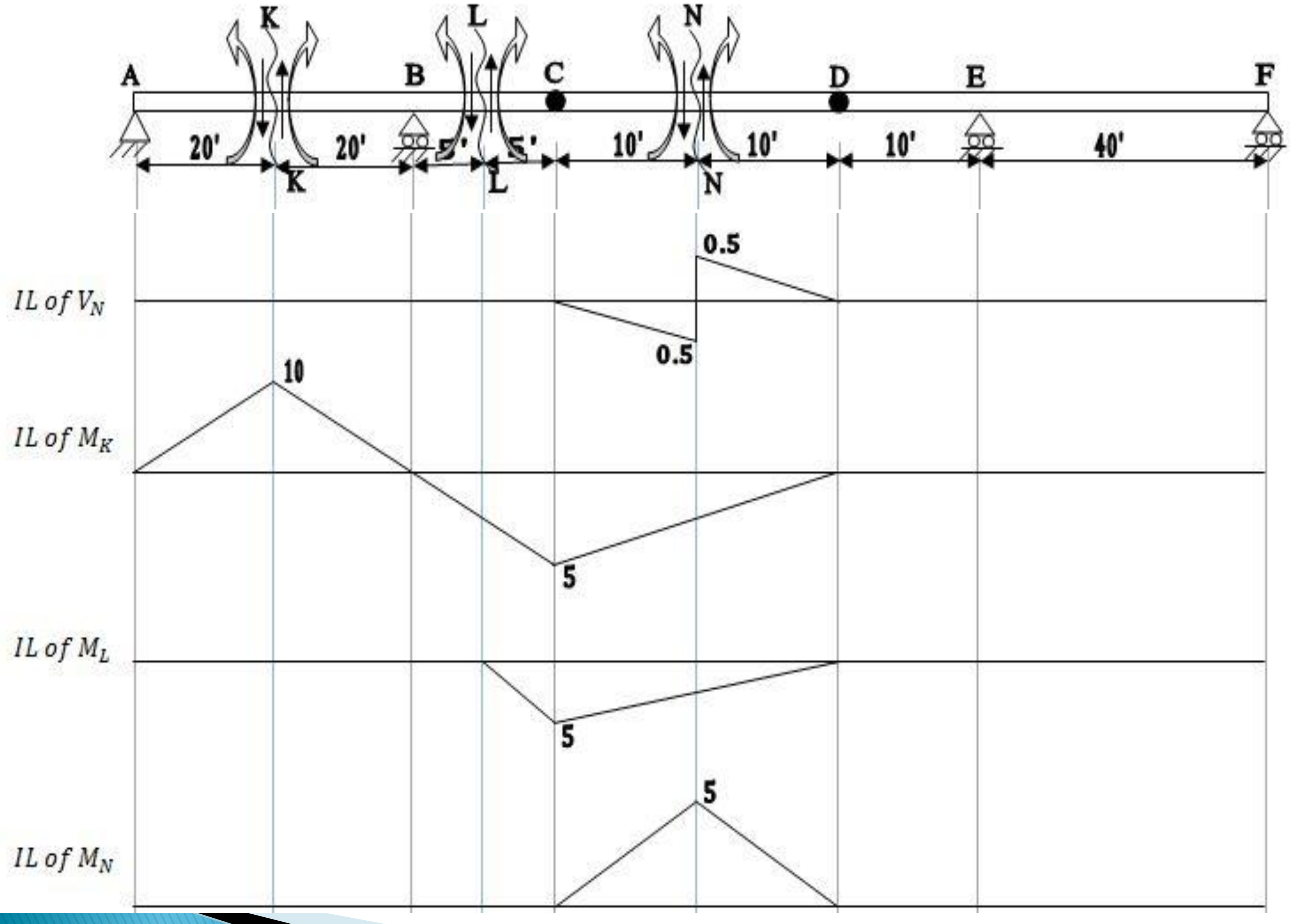


$\therefore \sum M_J = 0$ (+)

$2*1 + M_J = 0$

$\therefore M_J = -2$ KN-m


A
K
B
L
C
N
D
E
F
20'
20'
5'
5'
10'
10'
10'
40'
IL of R_A
1
0.25
IL of R_B
1
1.25
IL of R_E
1.25
1
IL of R_F
1
0.25
IL of V_K
0.5
0.5
0.25
IL of V_L
1

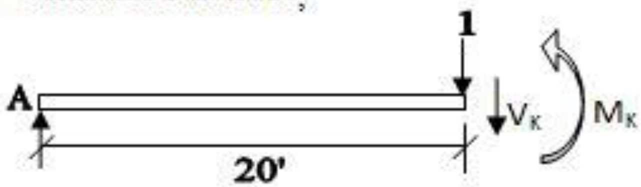


A
K
B
L
C
N
D
E
F
20'
20'
5'
5'
10'
10'
10'
40'
K
L
N
0.5
0.5
10
5
5
5

Calculation

IL of M_K

When load is at K,



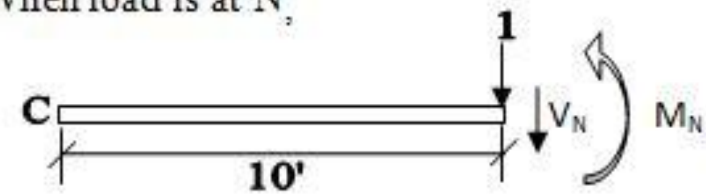
$\therefore \sum M_K = 0$ (+)

$0.5*20 - M_K = 0$

$\therefore M_K = 10$ kip-ft

IL of M_N

When load is at N,



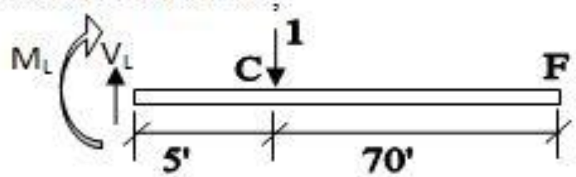
$\therefore \sum M_N = 0$ (+)

$0.5*10 - M_N = 0$

$\therefore M_N = 5$ kip-ft

IL of M_L

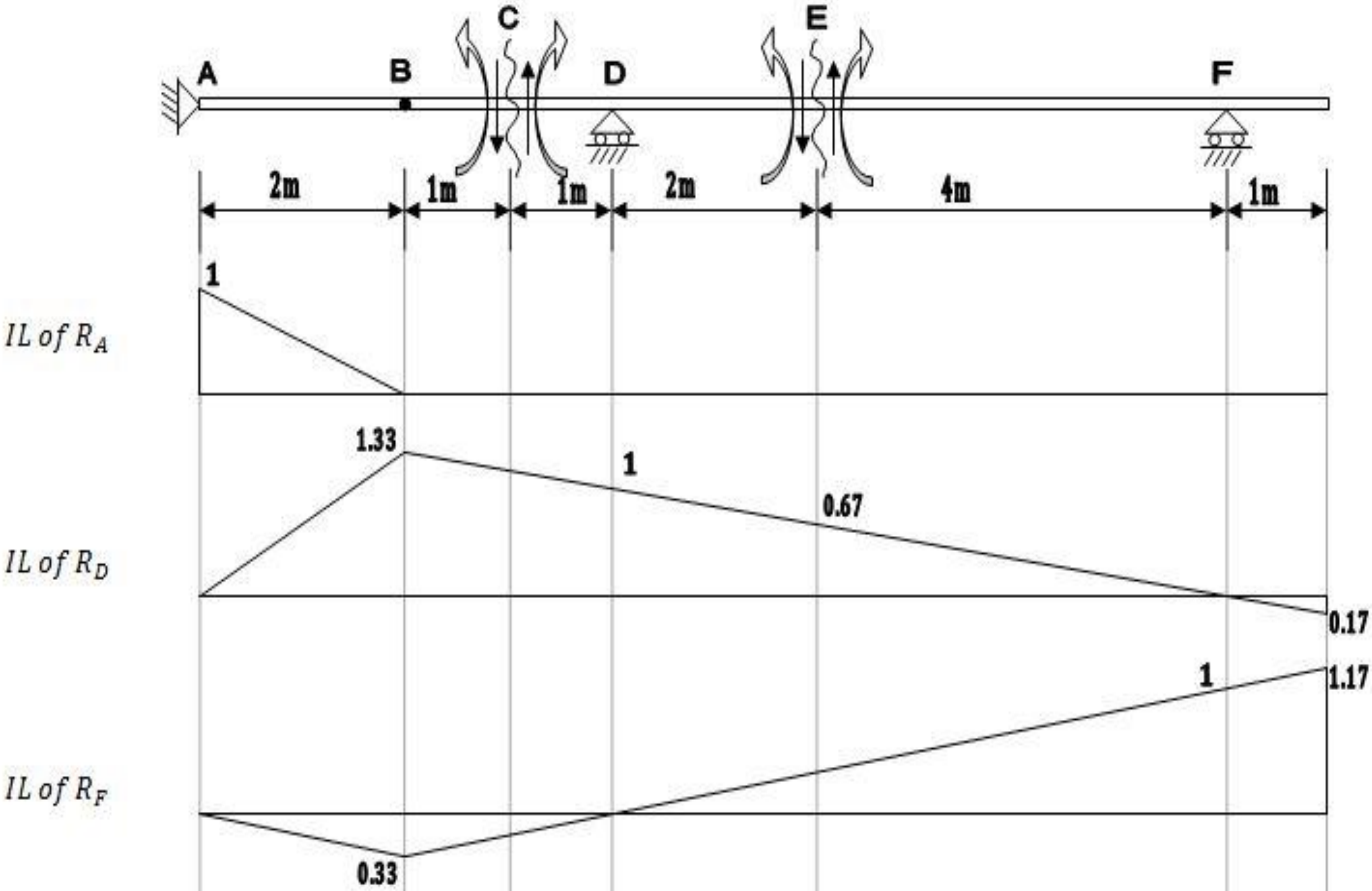
When load is at C,

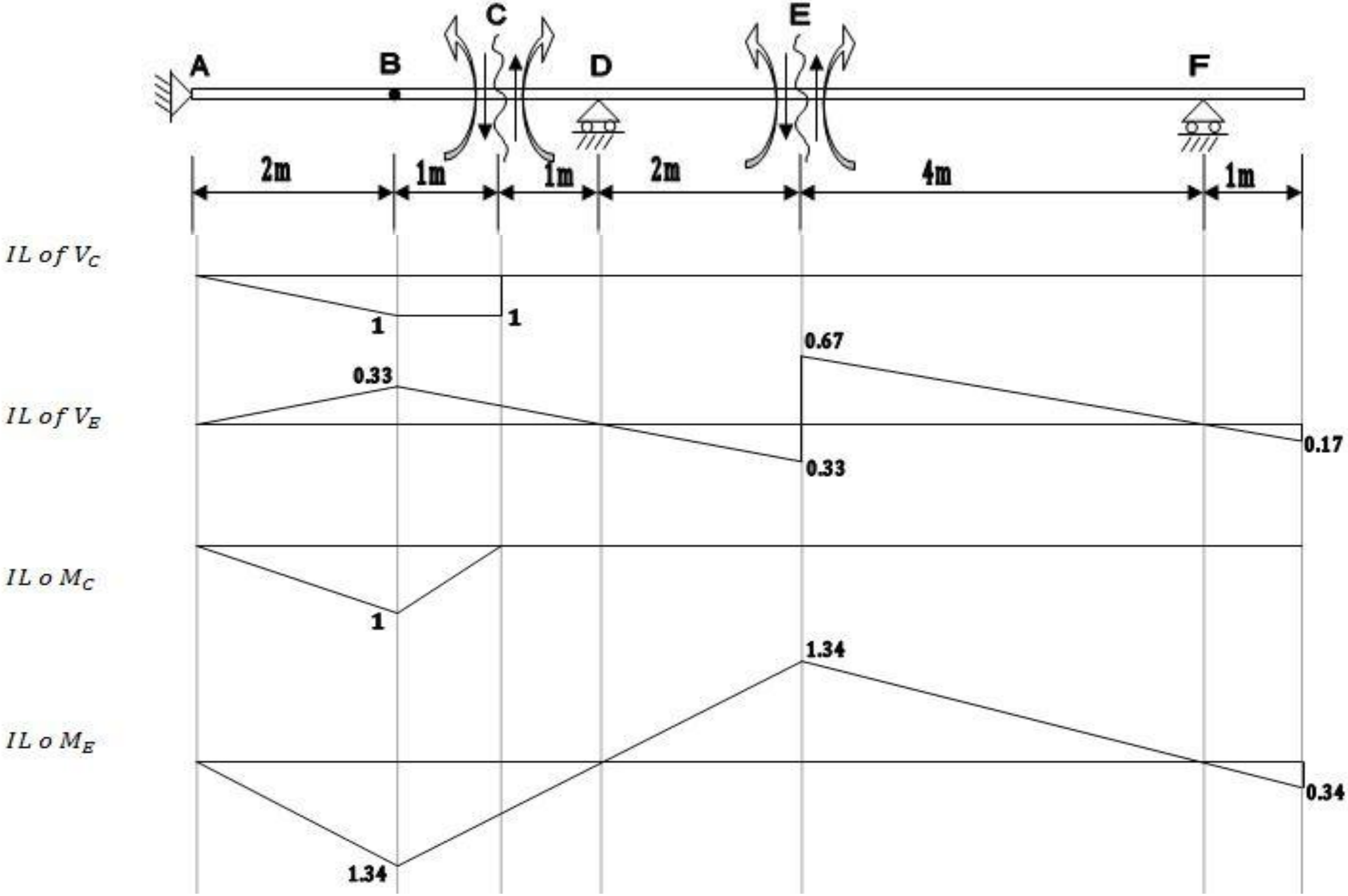


$\therefore \sum M_L = 0$ (+)

$5*1 + M_L = 0$

$\therefore M_L = -5$ kip-ft

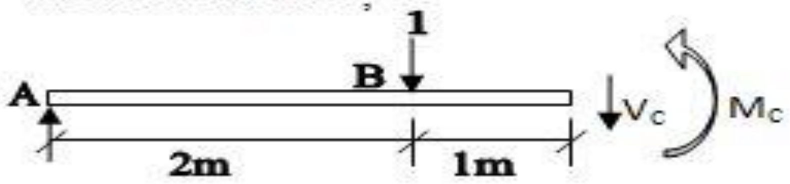

C
E
A
B
D
F
2m
1m
1m
2m
4m
1m
1
IL of R_A
1.33
1
0.67
IL of R_D
0.17
1
1.17
IL of R_F
0.33


A
B
C
D
E
F
2m
1m
1m
2m
4m
1m
$IL\ of\ V_C$
1
1
$IL\ of\ V_E$
0.33
0.67
0.33
0.17
$IL\ o\ M_C$
1
$IL\ o\ M_E$
1.34
1.34
0.34

Calculation

IL of M_C

When load is at B,



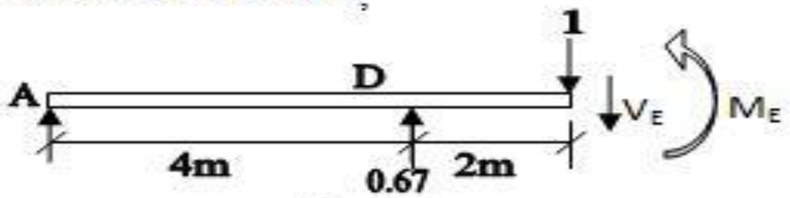
$\therefore \sum M_C = 0$ (↺ +)

$-1*1 - M_C = 0$

$\therefore M_C = -1$ KN-m

IL of M_E

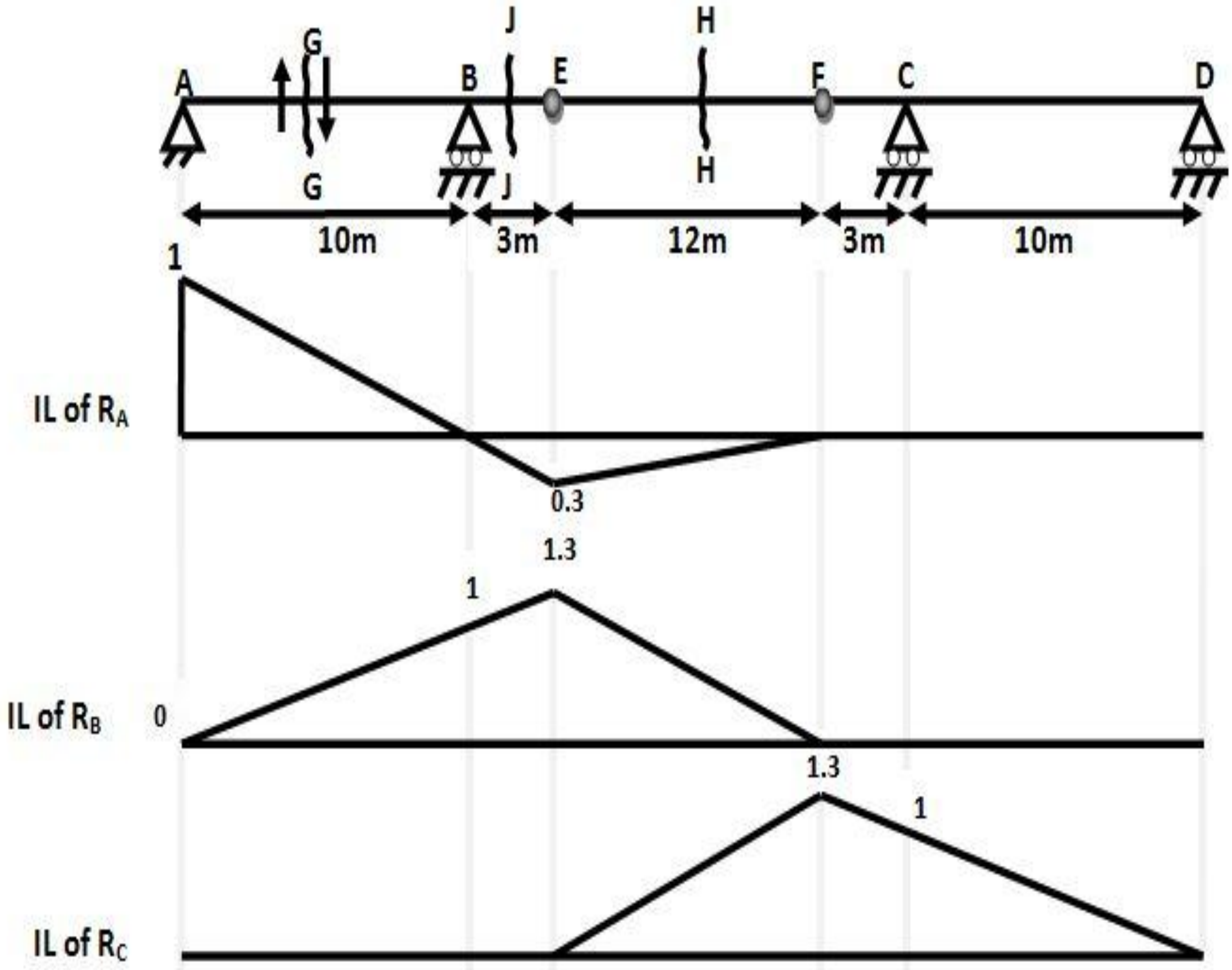
When load is at E,

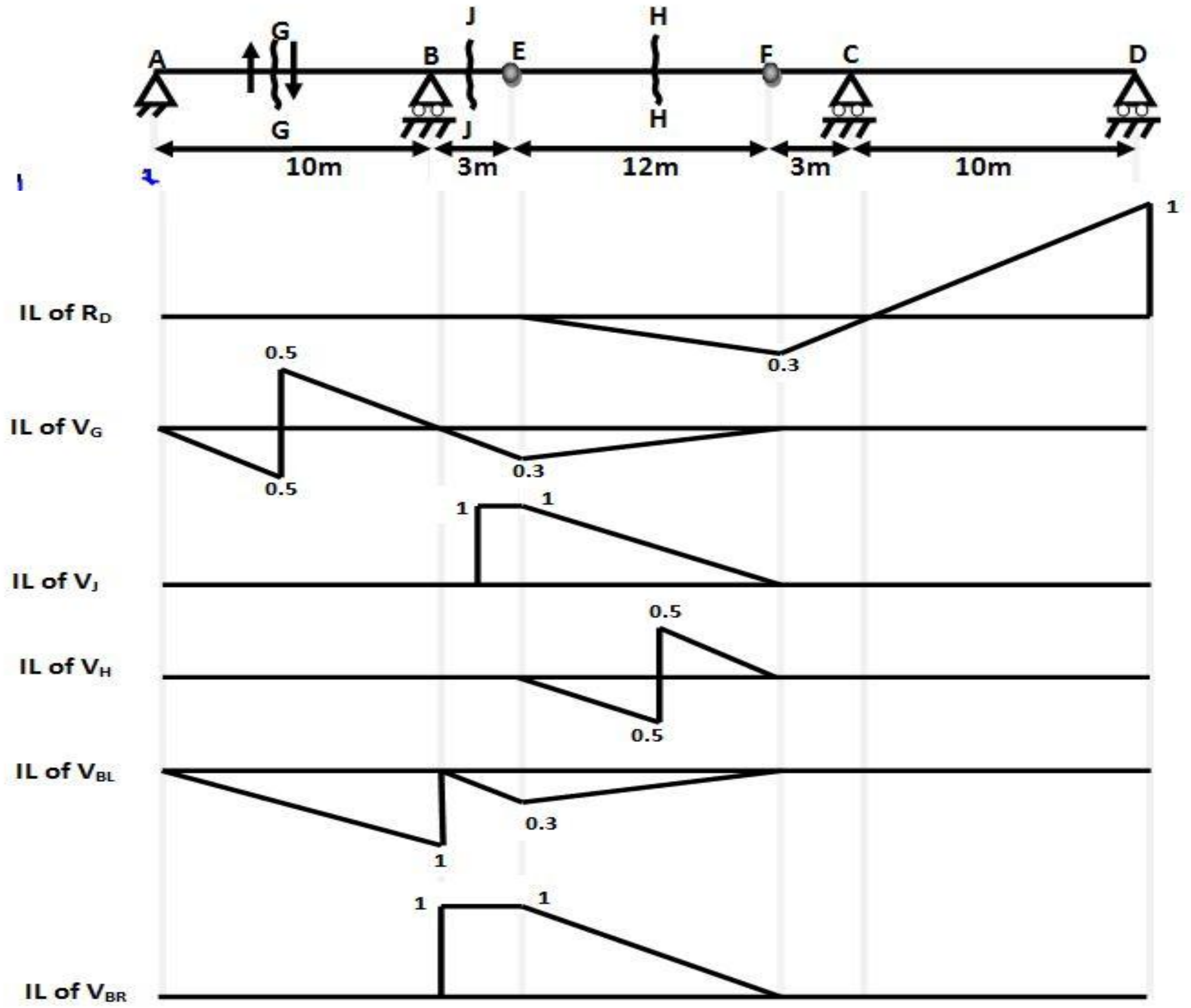


$\therefore \sum M_E = 0$ (↺ +)

$0.67*2 - M_E = 0$

$\therefore M_C = 1.34$ KN-m


G
J
H
A
B
E
F
C
D
G
J
H
10m
3m
12m
3m
10m
1
IL of R_A
0.3
1.3
1
IL of R_B
0
1.3
1
IL of R_C


A
G
G
B
J
J
E
H
H
F
C
D
10m
3m
12m
3m
10m
1
IL of R_D
0.3
0.5
IL of V_G
0.5
0.3
1
1
IL of V_J
0.5
IL of V_H
0.5
IL of V_{BL}
0.3
1
1
1
IL of V_{BR}

Calculation

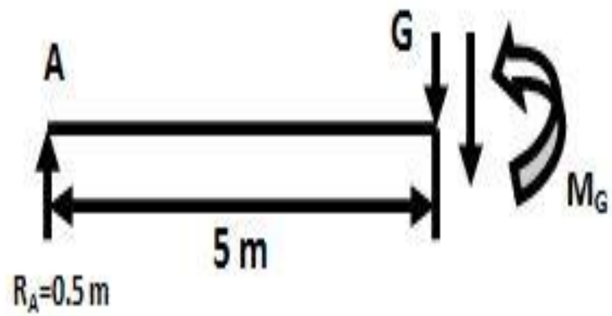
Moment at G – G

When load is at poin G

$$\sum M@G = 0\ (\curvearrowright +ve)$$

$$\Rightarrow 0.5 \times 5 - M_G = 0$$

$$\therefore M_G = 2.5$$



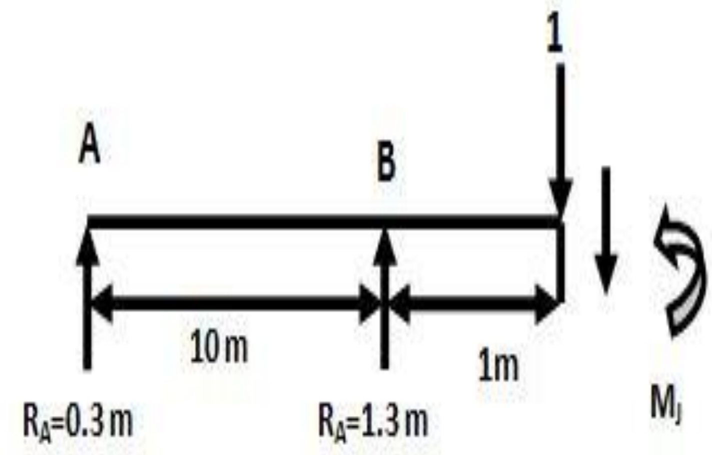
Moment at J – J

When load is at poin E

$$\sum M@J = 0\ (\curvearrowright +ve)$$

$$\Rightarrow 0.3 \times 11 + 1.3 \times 1 - M_J = 0$$

$$\therefore M_J = 2$$



Cont'd

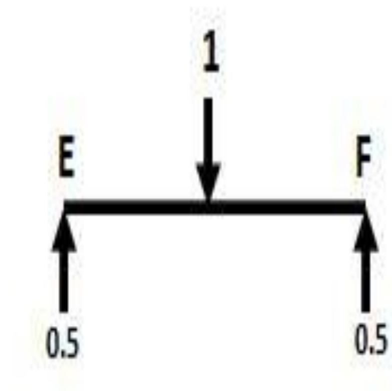
Moment at H – H

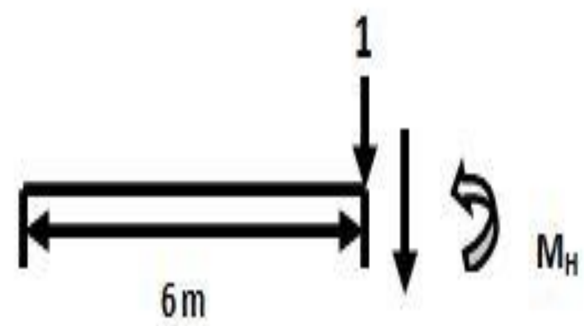
When load is at poin H

$$\sum M@H = 0\ (\curvearrowright +ve)$$

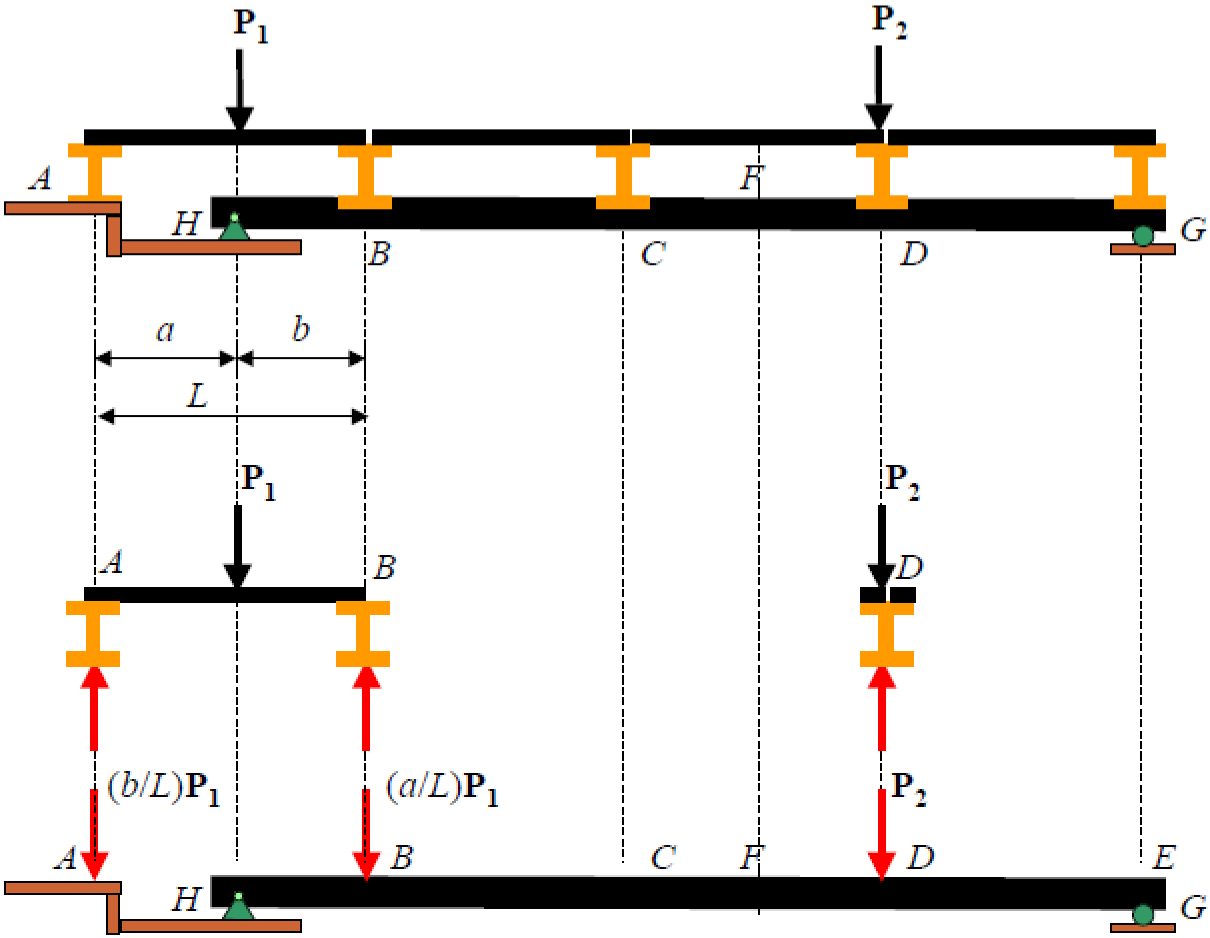
$$\Rightarrow 0.5 \times 6 - M_H = 0$$

$$\therefore M_H = 3$$



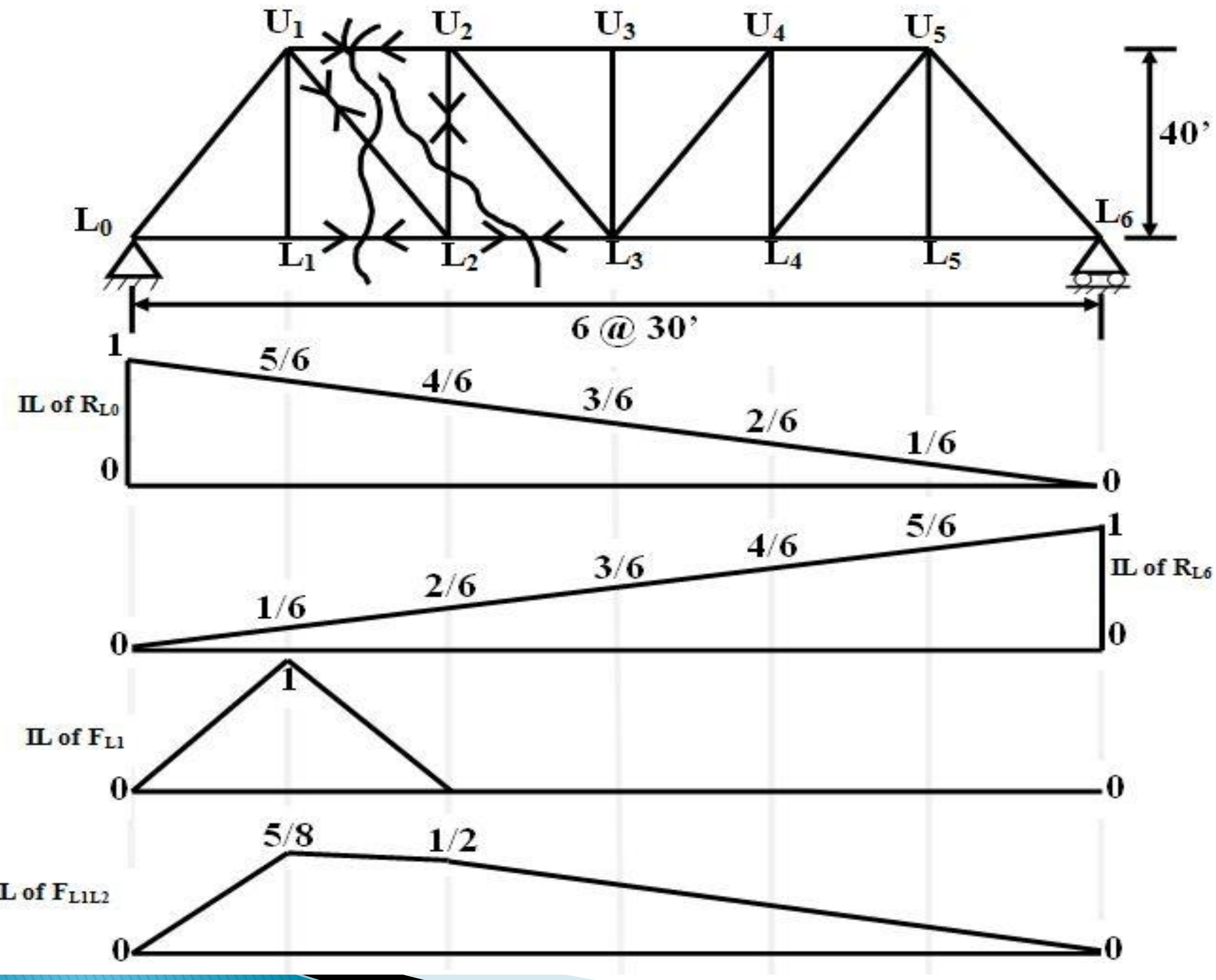


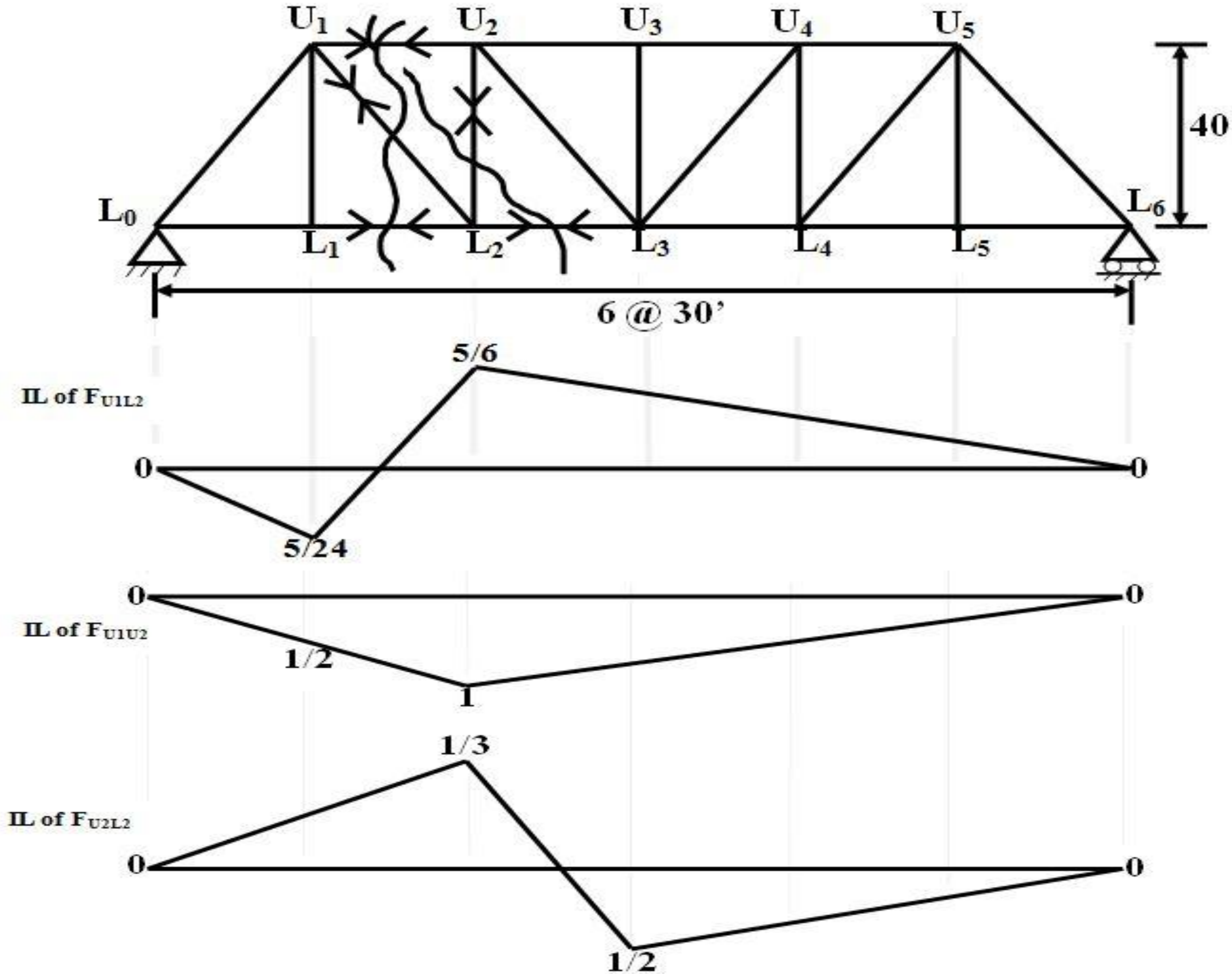
Influence Line of Girder with Floor Beams



More in PDF File (Influence line of girder & Truss).

Influence Lines of Truss




U1
U2
U3
U4
U5
40'
L0
L1
L2
L3
L4
L5
L6
6 @ 30'
5/6
IL of FU1L2
0
0
5/24
0
0
IL of FU1U2
1/2
1
1/3
IL of FU2L2
0
0
1/2

Calculation

Calculation of IL F_{L_1}

When Load at L_1

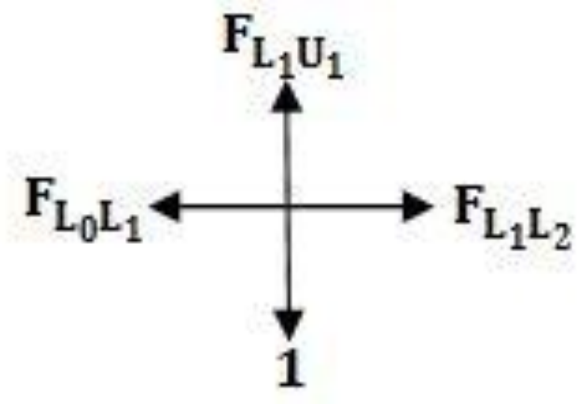
$$\sum V = 0 \ (\uparrow +ve)$$

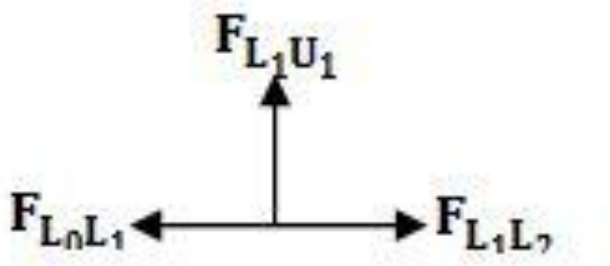
$$\therefore F_{L_1U_1} = 1$$

When Load at L_2 or L_0

$$\sum V = 0 \ (\uparrow +ve)$$

$$\therefore F_{L_1U_1} = 0$$





Calculation (Cont'd)

Calculation of IL $F_{L_1L_2}$

When Load at $L_0 - L_1$

$\sum M_{U_1} = 0$ (+ve)

$F_{L_1L_2} \times 40 = R_{L_6} \times (5 \times 30)$

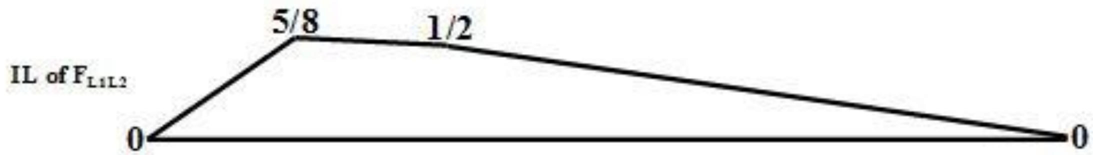
$\therefore F_{L_1L_2} = \frac{1}{6} \times \frac{150}{40} = \frac{5}{8}$

When Load at $L_2 - L_6$

$\sum M_{U_1} = 0$ (+ve)

$-F_{L_1L_2} \times 40 + R_{L_0} \times 30 = 0$

$\therefore F_{L_1L_2} = \frac{4}{6} \times \frac{30}{40} = \frac{1}{2}$



Calculation of IL $F_{U_1L_2}$

When Load at $L_0 - L_1$

$\sum V = 0$ (↑ +ve)

$F_{U_1L_2} \times \frac{4}{5} + R_{L_6} = 0$

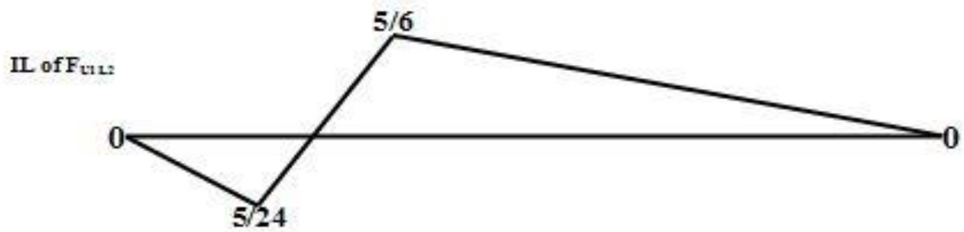
$\therefore F_{U_1L_2} = -\frac{5}{4} \times R_{L_6} = -\frac{5}{4} \times \frac{1}{6} = -\frac{5}{24}$

When Load at $L_2 - L_6$

$\sum V = 0$ (↑ +ve)

$-F_{U_1L_2} \times \frac{4}{5} + R_{L_0} = 0$

$\therefore F_{U_1L_2} = \frac{5}{4} \times R_{L_0} = \frac{5}{4} \times \frac{4}{6} = \frac{5}{6}$



Calculation (Cont'd)

Calculation of IL $F_{U_1U_2}$

When Load at $L_0 - L_1$

$\sum M_{L_2} = 0$ (+ve) ↻

$-F_{U_1U_2} \times 40 - R_{L_6} \times (4 \times 30) = 0$

$\therefore F_{U_1U_2} = -R_{L_6} \times \frac{(4 \times 30)}{40} = -\frac{1}{6} \times \frac{(4 \times 30)}{40} = -\frac{1}{2}$

When Load at $L_2 - L_6$

$\sum M_{L_2} = 0$ (+ve) ↻

$F_{U_1U_2} \times 40 + R_{L_0} \times 60 = 0$

$\therefore F_{U_1U_2} = -R_{L_0} \times \frac{60}{40} = -\frac{4}{6} \times \frac{60}{40} = -1$

Calculation of IL $F_{U_2L_2}$

When Load at $L_0 - L_2$

$\sum V = 0$ (↑ +ve)

$-F_{U_2L_2} + R_{L_6} = 0$

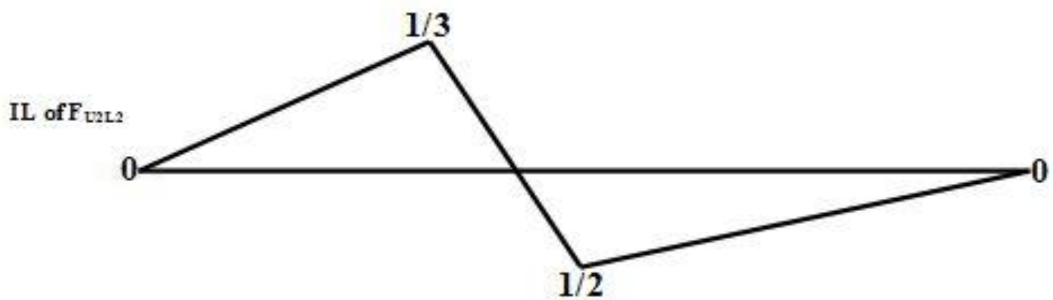
$\therefore F_{U_2L_2} = R_{L_6} = \frac{2}{6} = \frac{1}{3}$

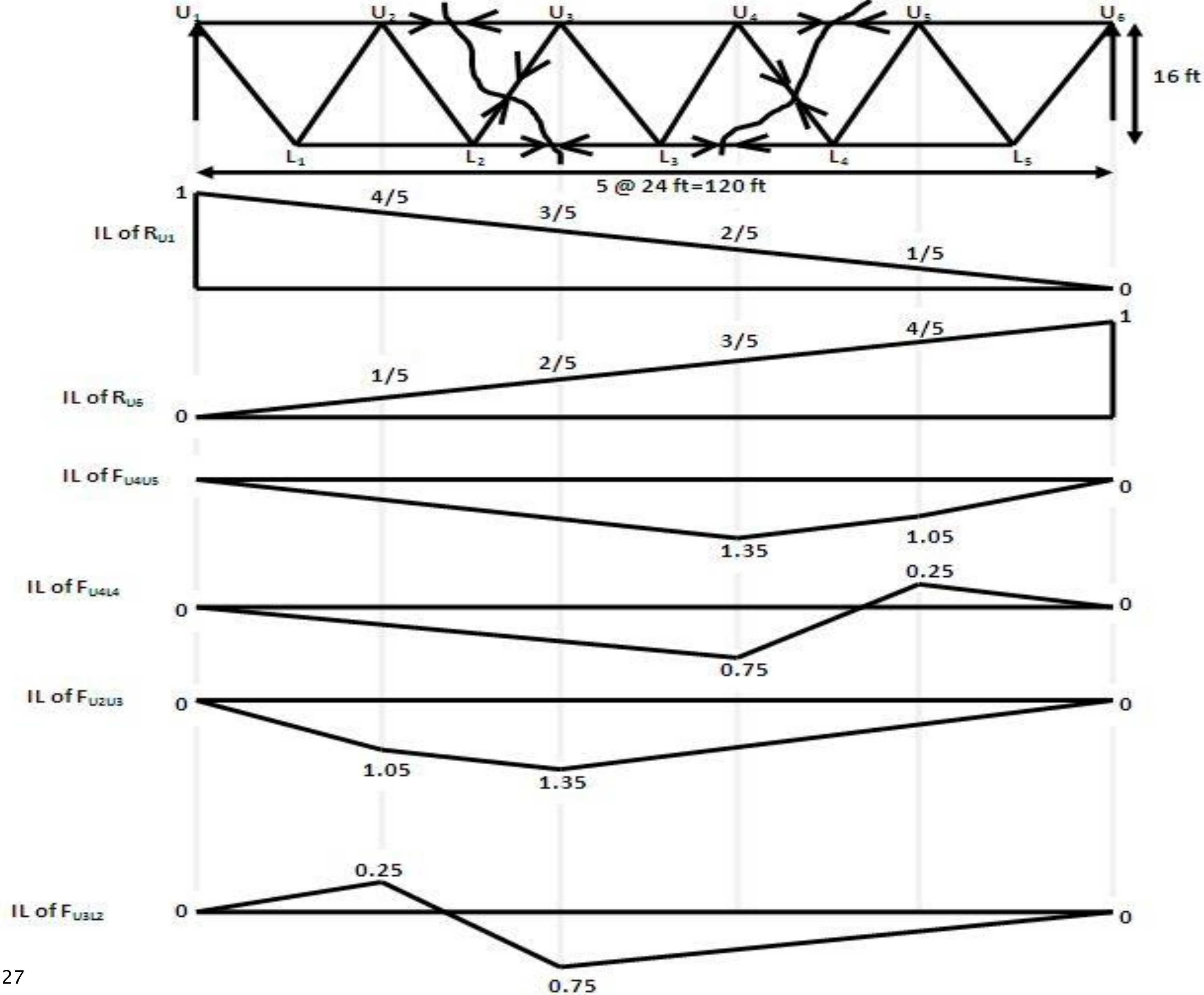
When Load at $L_3 - L_6$

$\sum V = 0$ (↑ +ve)

$F_{U_2L_2} + R_{L_0} = 0$

$\therefore F_{U_2L_2} = -R_{L_0} = -\frac{3}{6} = -\frac{1}{2}$




U1
U2
U3
U4
U5
U6
16 ft
L1
L2
L3
L4
L5
5 @ 24 ft=120 ft
IL of RU1
1
4/5
3/5
2/5
1/5
0
IL of RU6
0
1/5
2/5
3/5
4/5
1
IL of FU4U5
1.35
1.05
0
IL of FU4L4
0
0.75
0.25
0
IL of FU2U3
0
1.05
1.35
0
IL of FU3L2
0
0.25
0.75
0

Calculation

<u>Calculation of RL of</u> $F_{U_4U_5}$:

When load at $U_1 - U_4$:

$$\sum M_{L_4} = 0\ (\curvearrowright +ve)$$
$$\Rightarrow F_{U_4U_5} \times 16 - R_{U_6} \times 36 = 0$$
$$\Rightarrow F_{U_4U_5} = -\frac{36}{16} \times R_{U_6}$$
$$\Rightarrow F_{U_4U_5} = -\frac{36}{16} \times \frac{3}{5}$$
$$\therefore F_{U_4U_5} = -1.35$$

When load at $U_5 - U_6$:

$$\sum M_{L_4} = 0\ (\curvearrowright +ve)$$
$$\Rightarrow F_{U_4U_5} \times 16 + R_{U_1} \times 84 = 0$$
$$\Rightarrow F_{U_4U_5} = -\frac{84}{16} \times R_{U_1}$$
$$\Rightarrow F_{U_4U_5} = -\frac{36}{16} \times \frac{1}{5}$$
$$\therefore F_{U_4U_5} = -1.05$$

Calculation (Cont'd)

Calculation of RL of $F_{U_4L_4}$:

$When\ load\ at\ U_1 - U_4:$

$$\sum V = 0(\uparrow +ve)$$

$$\Rightarrow F_{U_4L_4} \times \frac{4}{5} + R_{U_6} = 0$$

$$\Rightarrow F_{U_4L_4} = \frac{5}{4} \times -R_{U_6}$$

$$\Rightarrow F_{U_4L_4} = -\frac{5}{4} \times \frac{3}{5}$$

$$\therefore F_{U_4L_4} = -0.75$$

$When\ load\ at\ U_5 - U_6:$

$$\sum V = 0(\uparrow +ve)$$

$$\Rightarrow -F_{U_4L_4} \times \frac{4}{5} + R_{U_1} = 0$$

$$\Rightarrow F_{U_4L_4} = \frac{5}{4} \times \frac{1}{5}$$

$$\therefore F_{U_4L_4} = 0.25$$

Calculation (Cont'd)

<u>Calculation of RL of</u> $F_{U_2U_3}$:

When load at $U_1 - U_2$:

$$\sum M_{L_2} = 0 \; (\curvearrowright +ve)$$

$$\Rightarrow F_{U_2U_3} \times 16 - R_{U_6} \times 84 = 0$$

$$\Rightarrow F_{U_2U_3} = -\frac{84}{16} \times \frac{1}{5}$$

$$\therefore F_{U_2U_3} = -1.05$$

When load at $U_3 - U_6$:

$$\sum M_{L_2} = 0 \; (\curvearrowright +ve)$$

$$\Rightarrow F_{U_2U_3} \times 16 + R_{U_1} \times 84 = 0$$

$$\Rightarrow F_{U_4U_5} = -\frac{36}{16} \times \frac{3}{5}$$

$$\therefore F_{U_2U_3} = -1.35$$

Calculation (Cont'd)

Calculation of RL of $F_{U_3L_2}$:

When load at $U_1 - U_2$:

$$\sum V = 0(\uparrow +ve)$$

$$\Rightarrow -F_{U_3L_2} \times \frac{4}{5} + R_{U_6} = 0$$

$$\Rightarrow F_{U_3L_2} = \frac{5}{4} \times \frac{1}{5}$$

$$\therefore F_{U_3L_2} = 0.25$$

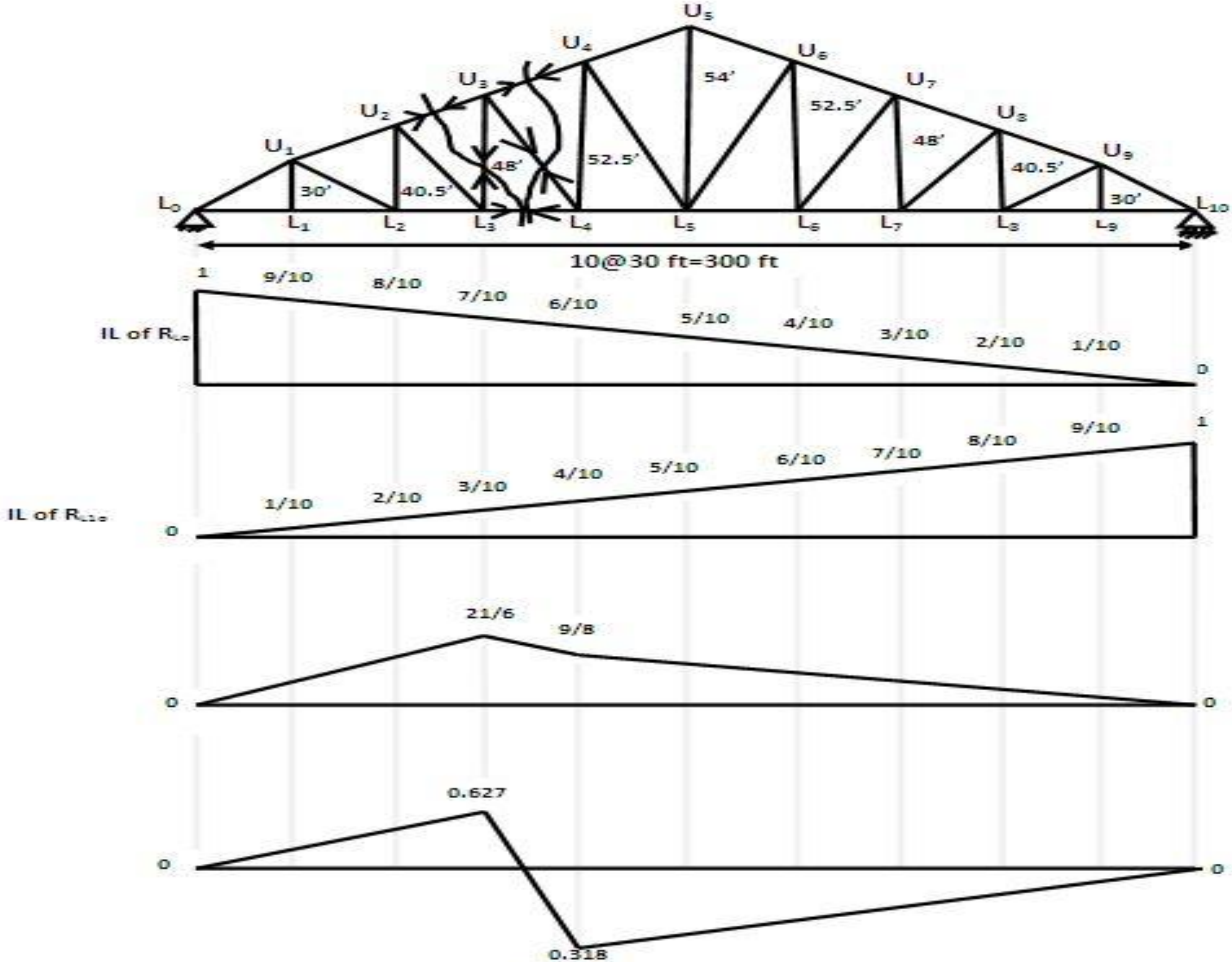
When load at $U_3 - U_6$:

$$\sum V = 0(\uparrow +ve)$$

$$\Rightarrow R_{U_1} + F_{U_3L_2} \times \frac{4}{5} = 0$$

$$\Rightarrow F_{U_3L_2} = -\frac{3}{5} \times \frac{5}{4}$$

$$\therefore F_{U_3L_2} = -0.75$$


U5
U4
U6
U3
U7
U2
U8
U1
U9
54'
52.5'
48'
40.5'
30'
L0
L1
L2
L3
L4
L5
L6
L7
L8
L9
L10
10@30 ft=300 ft
1
9/10
8/10
7/10
6/10
5/10
4/10
3/10
2/10
1/10
0
IL of R
1/10
2/10
3/10
4/10
5/10
6/10
7/10
8/10
9/10
1
IL of R
0
21/6
9/8
0
0
0.627
0
0
0.318

Calculation

$When\ load\ at\ \boldsymbol{L_0 - L_3}$

$$\sum M_{U_3} = 0 \ (\curvearrowright +ve)$$

$$\Rightarrow R_{L_3L_4} \times 48 - R_{40} \times (7 \times 30) = 0$$

$$\Rightarrow R_{L_3L_4} = R_{10} \times \frac{210}{48} = \frac{3}{10} \times \frac{210}{48} = \frac{21}{16}$$

$When\ load\ at\ \boldsymbol{L_4 - L_{10}}$

$$\sum M_{U_3} = 0 \ (\curvearrowright +ve)$$

$$\Rightarrow -R_{L_3L_4} \times 48 - R_{L_0} \times (3 \times 30) = 0$$

$$\Rightarrow R_{L_3L_4} = R_{L_0} \times \frac{90}{48} = \frac{6}{10} \times \frac{90}{48} = \frac{9}{8}$$

Calculation (Cont'd)

When load at $L_0 - L_3$

$$\sum M_G = 0\ (\curvearrowright +ve)$$

$$\Rightarrow -R_{40} \times (7 \times 30 \times 192) + R_{U_3L_3} \times 192 = 0$$

$$\Rightarrow R_{U_3L_3} = R_{L_{10}} \times \frac{402}{192}$$

$$\Rightarrow R_{U_3L_3} = \frac{3}{10} \times 2.09$$

$$\therefore R_{U_3L_3} = 0.628$$

When load at $L_4 - L_{10}$

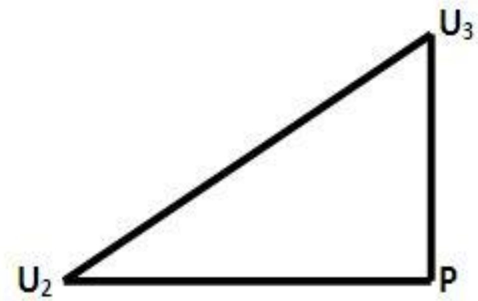
$$\sum M_G = 0\ (\curvearrowright +ve)$$

$$\Rightarrow -R_{L_0} \times (192 - 3 \times 30) - R_{U_3L_3}$$

$$\Rightarrow R_{U_3L_3} = -R_{L_0} \times \frac{102}{192}$$

$$\Rightarrow R_{U_3L_3} = -\frac{6}{10} \times 0.53$$

$$\therefore R_{U_3L_3} = -0.318$$



Here,

$U_2L_2 = 40.5'$

$U_3L_3 = 48'$

$U_3P = U_3L_3 - U_2L_2$

$\Rightarrow U_3P = 48 - 40.5$

$\therefore U_3P = 7.5'$

$\therefore U_2P = 30'$

Now, Similar Triangle formula

$$\frac{7.5}{30} = \frac{48}{x}$$

$\therefore x = 192$

More problems in PDF File (Influence line of girder & Truss).